

T.D. n°1 : Calcul vectoriel, fonctions à plusieurs variables, dérivées partielles.

Correction partielle

Exercice 6

1. $f(x, y, z) = x^2 + y^3 - 3z^2$

$Df = \mathbb{R}^3$ et

$$\frac{\partial f}{\partial x}(M) = 2x; \frac{\partial f}{\partial y}(M) = 3y^2; \frac{\partial f}{\partial z}(M) = -6z$$

2. $f(x, y, z, t) = (x^3 y^2 z^3 t, \cos(xyzt))$

$Df = \mathbb{R}^4$

$$\frac{\partial f}{\partial x}(M) = (3x^2 y^2 z^3 t, -\sin(xyzt) yzt) \quad \frac{\partial f}{\partial y}(M) = (2x^3 y z^3 t, -\sin(xyzt) xzt)$$

$$\frac{\partial f}{\partial z}(M) = (3x^3 y^2 z^2 t, -\sin(xyzt) xyt) \quad \frac{\partial f}{\partial t}(M) = (x^3 y^2 z^3, -\sin(xyzt) xyz)$$

3. $f(x, y, z) = x^2 y^3 (z + 3)^4$

$Df = \mathbb{R}^3$

$$\frac{\partial f}{\partial x}(M) = 2xy^3(z+3)^4; \frac{\partial f}{\partial y}(M) = 3x^2 y^2 (z+3)^4; \frac{\partial f}{\partial z}(M) = 4x^2 y^3 (z+3)^3$$

4. $f(x, y) = \ln(x^2 + 2y^2)$

$Df = \mathbb{R}^2 \setminus \{0; 0\}$

$$\frac{\partial f}{\partial x}(M) = \frac{2x}{x^2 + 2y^2}; \frac{\partial f}{\partial y}(M) = \frac{4y}{x^2 + 2y^2}$$

5. $f(x, y, z, t) = \frac{1}{x+y+z+t}$

$Df = \{(x, y, z, t) \in \mathbb{R}^4 / x + y + z + t \neq 0\}$

$$\frac{\partial f}{\partial x}(M) = \frac{-1}{(x+y+z+t)^2} = \frac{\partial f}{\partial y}(M) = \frac{\partial f}{\partial z}(M) = \frac{\partial f}{\partial t}(M)$$

6. $f(x, y, z) = (e^{xy} \sin^2(x+y), \frac{x}{y}, 5)$

$Df = \{(x, y, z) \in \mathbb{R}^3 / y \neq 0\}$

$$\frac{\partial f}{\partial x}(M) = \left(e^{xy} \sin(x+y) [y \cdot \sin(x+y) + 2\cos(x+y)], \frac{1}{y}, 0 \right)$$

$$\frac{\partial f}{\partial y}(M) = \left(e^{xy} \sin(x+y) [x \cdot \sin(x+y) + 2\cos(x+y)], \frac{-x}{y^2}, 0 \right) \text{ et } \frac{\partial f}{\partial z}(M) = (0, 0, 0)$$

7. $f(x, y, z) = \frac{xyz}{x^2 + y^2 + z^2}$ $Df = \mathbb{R}^3 \setminus \{0, 0, 0\}$

$$\frac{\partial f}{\partial x}(M) = \frac{(-x^2 + y^2 + z^2)yz}{(x^2 + y^2 + z^2)^2}; \frac{\partial f}{\partial y}(M) = \frac{(-y^2 + x^2 + z^2)xz}{(x^2 + y^2 + z^2)^2}; \frac{\partial f}{\partial z}(M) = \frac{(-z^2 + y^2 + x^2)xy}{(x^2 + y^2 + z^2)^2}$$

8. $f(x_1, x_2, \dots, x_{100}) = \prod_{i=1}^{100} x_i$ $Df = \mathbb{R}^{100}$

$$\frac{\partial f}{\partial x_i}(M) = \prod_{\substack{k=1 \\ k \neq i}}^{100} x_k$$

9. $f(u, v, w) = \frac{u+3v}{u^2-v}$ $Df = \{(u, v, w) \in \mathbb{R}^3 / u^2 \neq v\}$

$$\frac{\partial f}{\partial u}(M) = \frac{-(u^2 + 6uv + v)}{(u^2 - v)^2}; \frac{\partial f}{\partial v}(M) = \frac{u(3u + 1)}{(u^2 - v)^2}; \frac{\partial f}{\partial w}(M) = 0$$