Correction du TD sur les racines carrées

Exercice 1

$A = -\left(\sqrt{19}\right)^2$

$A = -\left(\sqrt{19} \times \sqrt{19}\right)$

$\boxed{A = -19}$

$B = \sqrt{32} \times \sqrt{2}$

$B = \sqrt{32 \times 2} = \sqrt{64}$

$\boxed{B = 8}$

$C = \sqrt{\dfrac{121}{144}} = \dfrac{\sqrt{121}}{\sqrt{144}}$

$\boxed{C = \dfrac{11}{12}}$

$D = \sqrt{36 + 64}$

$D = \sqrt{100}$

$\boxed{D = 10}$

Exercice 2

$E = \sqrt{8} - 2\sqrt{18} + \sqrt{32}$

$E = \sqrt{4 \times 2} - 2 \times \sqrt{9 \times 2} + \sqrt{16 \times 2}$

$E = \sqrt{4} \times \sqrt{2} - 2 \times \sqrt{9} \times \sqrt{2} + \sqrt{16} \times \sqrt{2}$

$E = 2 \times \sqrt{2} - 2 \times 3 \times \sqrt{2} + 4 \times \sqrt{2}$

$E = 2\sqrt{2} - 6\sqrt{2} + 4\sqrt{2}$

$E = (2 - 6 + 4)\sqrt{2}$

$E = (6 - 6)\sqrt{2}$

$E = 0\sqrt{2}$

$\boxed{E = 0}$

$F = \left(3\sqrt{2} - 5\right)\left(3\sqrt{2} + 5\right)$

$F = \left(3\sqrt{2}\right)^2 - (5)^2$

$F = 9 \times 2 - 25$

$F = 18 - 25$

$\boxed{F = -7}$

$G = \dfrac{\sqrt{80}}{3\sqrt{45}}$

$G = \dfrac{\sqrt{16 \times 5}}{3 \times \sqrt{9 \times 5}}$

$G = \dfrac{\sqrt{16} \times \sqrt{5}}{3 \times \sqrt{9} \times \sqrt{5}}$

$G = \dfrac{4\sqrt{5}}{9\sqrt{5}}$

$\boxed{G = \dfrac{4}{9}}$

Exercice 3

$H = \sqrt{81 - 49}$

$H = \sqrt{32}$

$H = \sqrt{16 \times 2}$

$H = \sqrt{16} \times \sqrt{2}$

$\boxed{H = 4\sqrt{2}}$

$I = \sqrt{300} + 4\sqrt{5}\sqrt{15}$

$I = \sqrt{100 \times 3} + 4 \times \sqrt{5} \times \sqrt{5 \times 3}$

$I = \sqrt{100} \times \sqrt{3} + 4 \times \sqrt{5} \times \sqrt{5} \times \sqrt{3}$

$I = 10\sqrt{3} + 4 \times 5 \times \sqrt{3}$

$I = 10\sqrt{3} + 20\sqrt{3}$

$\boxed{I = 30\sqrt{3}}$

Exercice 4

$J = \sqrt{15}\left(3 - \sqrt{15}\right) - \left(\sqrt{15} + 5\right)$

$J = \sqrt{15} \times 3 - \sqrt{15} \times \sqrt{15} - \sqrt{15} - 5$

$J = 3\sqrt{15} - 15 - \sqrt{15} - 5$

$\boxed{J = -20 + 2\sqrt{15}}$

$K = \left(\sqrt{3} - 2\sqrt{5}\right)^2$

$K = \left(\sqrt{3}\right)^2 - 2 \times \sqrt{3} \times 2 \times \sqrt{5} + \left(2\sqrt{5}\right)^2$

$K = 3 - 4 \times \sqrt{3 \times 5} + 4 \times 5$

$\boxed{K = 23 - 4\sqrt{15}}$

Exercice 5

A la calculatrice $\left(\sqrt{5} + \sqrt{3}\right) \div \left(\sqrt{8} + \sqrt{3}\right) \approx 0{,}870\ 110\ 291$ donc $\boxed{L \approx 0{,}87 \text{ à } 10^{-2} \text{ près}}$.

Exercice 6

- $KL^2 = \left(2\sqrt{11}\right)^2 = 4 \times 11 = 44$; $LM^2 = \left(\sqrt{154}\right)^2 = 154$; $KM^2 = \left(3\sqrt{22}\right)^2 = 9 \times 22 = 198$

  Test $\begin{cases} KM^2 = 198 \\ KL^2 + LM^2 = 44 + 154 = 198 \end{cases}$

  Donc $KM^2 = KL^2 + LM^2$; et d'après la réciproque du théorème de Pythagore le triangle KLM est rectangle en L.

- Puisque KLM est rectangle en L, $\mathcal{A}(KLM) = \dfrac{KL \times LM}{2} = \dfrac{2\sqrt{11} \times \sqrt{154}}{2} = \sqrt{11} \times \sqrt{11 \times 14}$

  Aire(KLM)= $\sqrt{11} \times \sqrt{11} \times \sqrt{14}$ et alors l'aire du triangle KLM vaut $\boxed{11\sqrt{14} \text{ cm}^2}$.

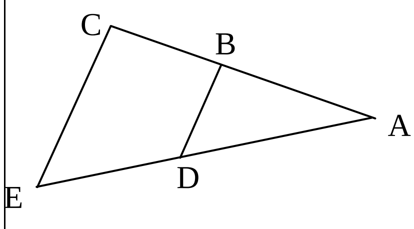
Exercice 7

Données $\begin{cases} \text{Les point A, B, C et A, D, E sont alignés sur deux droites sécantes en A} \\ \text{Les droites (BD) et (CE) sont parallèles.} \end{cases}$

On peut utiliser le théorème de Thalès :

$\dfrac{AB}{AC} = \dfrac{AD}{AE} = \dfrac{BD}{CE}$; en remplaçant on obtient : $\dfrac{6}{6+3} = \dfrac{AD}{\sqrt{45}}$ et ensuite $AD = \dfrac{6 \times \sqrt{45}}{9} = \dfrac{6 \times \sqrt{9} \times \sqrt{5}}{9}$

$AD = \dfrac{6 \times 3 \times \sqrt{5}}{9} = \dfrac{18\sqrt{5}}{9}$ et alors $\boxed{AD = 2\sqrt{5} \text{ cm}}$.



Exercice 8

1/ $A = 3x^2 - 2x + 1$.

$x = \sqrt{2}$; $A = 3(\sqrt{2})^2 - 2\sqrt{2} + 1 = 3\times2 - 2\sqrt{2} + 1 = \boxed{7 - 2\sqrt{2}}$.

$x = 3\sqrt{2}$; $A = 3(3\sqrt{2})^2 - 2(3\sqrt{2}) + 1 = 3\times9\times2 - 6\sqrt{2} + 1 = \boxed{55 - 6\sqrt{2}}$.

$x = -\sqrt{2}$; $A = 3\times(-\sqrt{2})^2 - 2\times(-\sqrt{2}) + 1 = 3\times2 + 2\sqrt{2} + 1 = \boxed{7 + 2\sqrt{2}}$.

$x = \frac{\sqrt{2}}{3}$; $A = 3(\frac{\sqrt{2}}{3})^2 - 2\times\frac{\sqrt{2}}{3} + 1 = 3\times\frac{2}{9} - \frac{2\sqrt{2}}{3} + 1 = \frac{2}{3} - \frac{2\sqrt{2}}{3} + 1 = \boxed{\frac{5}{3} - \frac{2\sqrt{2}}{3}}$ $\left(= \frac{5 - 2\sqrt{2}}{3}\right)$.

$x = -\frac{\sqrt{2}}{3}$; $A = 3(-\frac{\sqrt{2}}{3})^2 - 2\times(-\frac{\sqrt{2}}{3}) + 1 = 3\times\frac{2}{9} + \frac{2\sqrt{2}}{3} + 1 = \frac{2}{3} + \frac{2\sqrt{2}}{3} + 1 = \boxed{\frac{5}{3} + \frac{2\sqrt{2}}{3}}$ $\left(= \frac{5 + 2\sqrt{2}}{3}\right)$.

2/ $B = (3x - 1)^2 - (x + 2)^2$.

a/ Pour $x = \sqrt{5}$, $B = (3\sqrt{5} - 1)^2 - (\sqrt{5} + 2)^2$

$B = (3\sqrt{5})^2 - 2\times3\sqrt{5}\times1 + 1^2 - (\sqrt{5})^2 + 2\times\sqrt{5}\times2 + 2^2$.

Donc si $x = \sqrt{5}$, $B = 9\times5 - 6\sqrt{5} + 1 - (5 + 4\sqrt{5} + 4) = 45 - 6\sqrt{5} + 1 - 5 - 4\sqrt{5} - 4$.

Finalement pour $x = \sqrt{5}$, $\boxed{B = 37 - 10\sqrt{5}}$.

b/ $B = [(3x - 1) - (x + 2)]\,[(3x - 1) + (x + 2)] = (3x - 1 - x - 2)(3x - 1 + x + 2)$.

Donc $\boxed{B = (2x - 3)(4x + 1)}$ et pour $x = \sqrt{5}$ on a :

$B = (2\sqrt{5} - 3)(4\sqrt{5} + 1) = 2\sqrt{5}\times4\sqrt{5} + 2\sqrt{5} - 3\times4\sqrt{5} - 3$

$B = 8\times5 + 2\sqrt{5} - 12\sqrt{5} - 3 = \boxed{37 - 10\sqrt{5}}$

Exercice 9

On pose $a = \sqrt{181 + 52\sqrt{3}}$ et $b = \sqrt{181 - 52\sqrt{3}}$.

1/ a/ $181 - 52\sqrt{3} \approx 90{,}9 > 0$.

b/ b existe bien puisque c'est la racine carrée d'un nombre positif : $181 - 52\sqrt{3}$.

2/ a/ $\boxed{a^2 = \left(\sqrt{181 + 52\sqrt{3}}\right)^2 = 181 + 52\sqrt{3}}$ et $\boxed{b^2 = \left(\sqrt{181 - 52\sqrt{3}}\right)^2 = 181 - 52\sqrt{3}.}$

$ab = \sqrt{181 + 52\sqrt{3}} \times \sqrt{181 - 52\sqrt{3}} = \sqrt{(181 + 52\sqrt{3})(181 - 52\sqrt{3})} = \sqrt{181^2 - (52\sqrt{3})^2}$.

Donc $\boxed{ab = \sqrt{32761 - 2704\times3} = \sqrt{24649} = 157}$.

b/ $(a + b)^2 = a^2 + 2ab + b^2 = 181 + 52\sqrt{3} + 2\times157 + 181 - 52\sqrt{3} = 181 + 314 + 181 = 676$.

donc $\boxed{a + b = \sqrt{676} = 26}$.

3/ a/ $(13 + 2\sqrt{3})^2 = 13^2 + 2\times13\times2\sqrt{3} + (2\sqrt{3})^2 = 169 + 52\sqrt{3} + 4\times3 = 169 + 12 + 2\sqrt{3} = 181 + 52\sqrt{3}$

donc $\boxed{a = \sqrt{(13 + 2\sqrt{3})^2} = 13 + 2\sqrt{3}}$ car $\underline{13 + 2\sqrt{3} > 0}$.

b/ $(13 - 2\sqrt{3})^2 = 13^2 - 2\times13\times2\sqrt{3} + (2\sqrt{3})^2 = 169 - 52\sqrt{3} + 4\times3 = 169 + 12 - 2\sqrt{3} = \boxed{181 - 52\sqrt{3}}$

donc $\boxed{b = \sqrt{(181 - 2\sqrt{3})^2} = 13 - 2\sqrt{3}}$ car $\underline{13 - 2\sqrt{3} > 0}$.

c/ On a : $\boxed{a + b = 13 + 2\sqrt{3} + 13 - 2\sqrt{3} = 26}$.