T.D. n°1 : Calcul vectoriel, fonctions à plusieurs variables, dérivées partielles.

Correction partielle

Exercice 6

1. $\boldsymbol{f(x,y,z) = x^2 + y^3 - 3z^2}$ $\quad Df = \mathbb{R}^3$ *et*

$$\frac{\partial f}{\partial x}(M) = 2x \;;\; \frac{\partial f}{\partial y}(M) = 3y^2 \;;\; \frac{\partial f}{\partial z}(M) = -6z$$

2. $\boldsymbol{f(x,y,z,t) = (x^3y^2z^3t\,, \cos(xyzt))}$ $\quad Df = \mathbb{R}^4$

$$\frac{\partial f}{\partial x}(M) = (\,3x^2y^2z^3t\,, -\sin(xyzt)yzt) \qquad \frac{\partial f}{\partial y}(M) = (\,2x^3yz^3t\,, -\sin(xyzt)xzt)$$

$$\frac{\partial f}{\partial z}(M) = (\,3x^3y^2z^2t\,, -\sin(xyzt)xyt) \qquad \frac{\partial f}{\partial t}(M) = (\,x^3y^2z^3\,, -\sin(xyzt)xyz)$$

3. $\boldsymbol{f(x,y,z) = x^2y^3(z+3)^4}$ $\quad Df = \mathbb{R}^3$

$$\frac{\partial f}{\partial x}(M) = 2xy^3(z+3)^4 \;;\; \frac{\partial f}{\partial y}(M) = 3x^2y^2(z+3)^4 \;;\; \frac{\partial f}{\partial z}(M) = 4\,x^2y^3(z+3)^3$$

4. $\boldsymbol{f(x,y) = \ln(x^2 + 2y^2)}$ $\quad Df = \mathbb{R}^2\backslash\{0;0\}$

$$\frac{\partial f}{\partial x}(M) = \frac{2x}{x^2+2y^2} \;;\; \frac{\partial f}{\partial y}(M) = \frac{4y}{x^2+2y^2}$$

5. $\boldsymbol{f(x,y,z,t) = \frac{1}{x+y+z+t}}$ $\quad Df = \{(x,y,z,t) \in \mathbb{R}^4 / \; x+y+z+t \neq 0\}$

$$\frac{\partial f}{\partial x}(M) = \frac{-1}{(x+y+z+t)^2} = \frac{\partial f}{\partial y}(M) = \frac{\partial f}{\partial z}(M) = \frac{\partial f}{\partial t}(M)$$

6. $\boldsymbol{f(x,y,z) = (\,e^{xy}\sin^2(x+y), \frac{x}{y}\,, 5)}$ $\quad Df = \{(x,y,z) \in \mathbb{R}^3 \,/\, y \neq 0\}$

$$\frac{\partial f}{\partial x}(M) = \left(e^{xy}\sin(x+y)\;\;[y.\sin(x+y) + 2\cos(x+y)\,]\,, \frac{1}{y}\,, 0\right)$$

$$\frac{\partial f}{\partial y}(M) = \left(e^{xy}\sin(x+y)\;\;[x.\sin(x+y) + 2\cos(x+y)\,]\,, \frac{-x}{y^2}\,, 0\right) \text{ et } \frac{\partial f}{\partial z}(M) = (\,0,0,0)$$

7. $\boldsymbol{f(x,y,z) = \frac{xyz}{x^2+y^2+z^2}}$ $\quad Df = \mathbb{R}^3\backslash\{0,0,0\}$

$$\frac{\partial f}{\partial x}(M) = \frac{(-x^2+y^2+z^2)yz}{(x^2+y^2+z^2)^2} \qquad \frac{\partial f}{\partial y}(M) = \frac{(-y^2+x^2+z^2)xz}{(x^2+y^2+z^2)^2} \qquad \frac{\partial f}{\partial z}(M) = \frac{(-z^2+y^2+x^2)xy}{(x^2+y^2+z^2)^2}$$

8. $\boldsymbol{f(x_1, x_2, \ldots, x_{100}) = \prod_{i=1}^{100} x_i}$ $\quad Df = \mathbb{R}^{100}$

$$\frac{\partial f}{\partial x_i}(M) = \prod_{\substack{k=1 \\ k\neq i}}^{100} x_k$$

9. $\boldsymbol{f(u,v,w) = \frac{u+3v}{u^2-v}}$ $\quad Df = \{(u,v,w) \in \mathbb{R}^3 / \; u^2 \neq v\}$

$$\frac{\partial f}{\partial u}(M) = \frac{-(u^2+6uv+v)}{(u^2-v)^2} \;;\; \frac{\partial f}{\partial v}(M) = \frac{u(3u+1)}{(u^2-v)^2} \;;\; \frac{\partial f}{\partial w}(M) = 0$$

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